Global Nonlinear Model Identification with Multivariate Splines

PROEFSCHRIFT

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Summary

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A model is an abstraction of physical reality in which mathematics are used to reduce its complexity into a conceptual structure. The field of science concerned with the identification of models of physical systems is called *system identification*. In this thesis, a new methodology is proposed for the identification of models of nonlinear systems with complex dynamics using *multivariate simplex splines*. Modeling systems with nonlinear dynamics is a challenging task, and currently only a handful of methods exist that are capable of creating sufficiently accurate models of such systems. The four most widely known of these methods are neural networks, kernel methods, polynomial blending methods, and spline methods. All these methods are able to produce models of an arbitrarily high approximation power on a global model scale. Until recently, however, all these methods suffered from inherent shortcomings. Neural networks are essentially black-box models and use global basis functions, resulting in complex, nontransparent, and inefficient computational schemes for their training and evaluation. Kernel methods are non-parametric in nature, which means that in principle there are as many kernel functions as there are data points, leading to inefficient computational schemes for large datasets. Polynomial blending methods use fuzzy logic techniques to blend local polynomial models into a single global model. The tuning of the fuzzy blending operation is done based on expert knowledge, with the result that it is unlikely to ever become a fully automated technique. Polynomial spline methods have been successfully used in the past for the modeling of nonlinear systems. However, these spline methods employed multivariate tensor product B-splines which are limited to rectangular domains, and are incapable of fitting widely occurring scattered data.

The new methodology proposed in this thesis is based on multivariate simplex splines, which are a recent type of multivariate spline that have a number of important advantages over the above mentioned methods. Firstly, simplex splines have a local polynomial ba-

sis, which implies that only small subsets of parameters and basis polynomials need to be considered during estimation and evaluation, resulting in efficient computational schemes. Secondly, simplex spline models are parametric models, which allows for efficient approximation of very large datasets. Thirdly, the simplex splines are linear in the parameters, meaning that linear regression methods can be used for their estimation. Fourthly, the simplex splines are defined on non-rectangular domains and can be used to approximate scattered data. And finally, the quality of simplex spline based models can be assessed using a number of unique and powerful model quality assessment methods.

Multivariate simplex splines consist of polynomial basis functions, called B-form polynomials, which are defined on geometric structures called simplices. Every simplex supports a single B-form polynomial which itself consists of a linear combination of *Bernstein* basis polynomials. Each individual Bernstein basis polynomial is scaled by a single coefficient called a B-coefficient. The B-coefficients have a special property in the sense that they have a unique spatial location inside their supporting simplex. This spatial structure, also known as the B-net, provides a number of unique capabilities that add to the desirability of the simplex splines as a tool for data approximation. For example, the B-net simplifies local model modification by directly relating specific model regions to subsets of B-coefficients involved in shaping the model in those regions. This particular capability has the potential to play an important role in future *adaptive model based control systems*. In such a control system, an on-board simplex spline model can be locally adapted in real time to reflect changes in system dynamics.

The approximation power of the multivariate simplex splines can be increased by joining any number of simplices together into a geometric structure called a triangulation. Triangulations come in many shapes and sizes, ranging from configurations consisting of just two simplices to configurations containing millions of simplices. Triangulations can be optimized by locally increasing or decreasing the density of simplices to reflect local system complexity. In principle, the total number of simplices in a triangulation is bounded only by the available computational resources. This thesis shows, however, that there is an important practical limit to the size and resolution of a triangulation. This practical limit is the result of every simplex requiring a minimum data content which is determined by the degree and continuity order of the basis polynomials. It was shown in this thesis that this *data coverage problem* requires a new approach towards triangulation optimization, as methods in the existing literature do not consider per-simplex data coverage as an optimization parameter. The newly proposed method for triangulation optimization produces triangulations that are specifically suited for use with simplex splines by ensuring that every individual simplex in a triangulation contains a minimum amount of data.

While multivariate simplex splines have been used in the past to model scattered nonlinear data in two and three dimensions, no methodology existed for their use inside a framework for system identification. The unique properties of the simplex splines, together with the above mentioned advantages over existing data approximators, makes them highly desirable for use within such a framework. It is the main objective of this thesis to present a new methodology for system identification based on multivariate simplex splines. This new methodology encompasses the three main aspects of system identification; model structure selection, parameter estimation, and model validation. The aspect of model structure selection for the multivariate simplex splines consists of two parts. The first part is the geometric model structure selection which consists of the selection of the spline model dimensions and the creation of a triangulation embedded in this set of dimensions. The second part is the determination of the polynomial model structure. For the aspect of parameter estimation, a new formulation of the standard linear regression model structure was developed. In this formulation, the B-form polynomials of the simplex splines form the regressors. Using the new regression model structure, a number of different parameter estimation techniques can be employed to estimate the B-coefficients of the B-form polynomials. This thesis introduces two such methods for parameter estimation. The first is a generalized least squares estimator, which enables the estimation of B-coefficients on simplices containing measurement noise of varying magnitudes. The second parameter estimator is a differentially constrained recursive least squares estimator which allows, in real-time, the reconfiguration of spline models using incoming observations. During the aspect of model validation, the quality of the estimated spline models is assessed using existing methods based on an analysis of model residuals and parameter variances. Additionally, a number of completely new quality assessment methods are enabled by the use of the B-form polynomials. For example, the variances of the B-coefficients can be pinpointed to specific locations within the model. This means that regions of high parameter variance can be isolated within the global model and subjected to further analysis. These unique and powerful properties together may result in a new perspective on system identification and parameter estimation, potentially leading to further innovations in the field.

This thesis introduces three major theoretical innovations in the field of multivariate spline theory. These innovations were essential in the creation of an effective method for system identification with simplex splines. The first of these innovations was the definition of the differential constraints, which are used to constrain the directional derivatives of the simplex splines at selected locations within the spline domain. The differential constraints enable bounded model extrapolation and limit polynomial divergence near the bounds of the spline domain. Additionally, the differential constraints can be applied to impose boundary conditions like *Dirichlet* or *Cauchy* conditions on the simplex spline functions, thereby enabling the approximation of solutions to boundary value problems using simplex splines. The second innovation was the development of a theory for the quantification of B-net propagation, a new effect observed in large scale triangulations. B-net propagation is the spreading of local disturbances from the B-net of one simplex to that of its neighbors. It was proved that B-net propagation effectively transforms a simplex spline function from a local approximator into an global approximator if its continuity order is high with respect to its polynomial degree, and when it is defined on the most widely used triangulation type.

A final innovation was a new formulation of the B-form in global Cartesian coordinates instead of local barycentric coordinates. The Bernstein basis polynomials of the simplex splines are functions in terms of local barycentric coordinates, which means that their global interpretation is meaningless. The new formulation of the B-form polynomials in global coordinates adds a global interpretation capability. Additionally, and more importantly, the new formulation enables the optimization of triangulation and B-coefficients in a single step, thereby avoiding the need for separate triangulation optimization.

Aircraft aerodynamics are notoriously nonlinear, and the identification of accurate aerodynamic models from flight data has historically been a challenging task. Aerodynamic models are crucial in the correct functioning of flight simulators and flight control systems. The higher the quality of an aerodynamic model, the more accurate its predictions on the real aerodynamic forces and moments acting on an aircraft. For flight simulator applications, this directly translates into an increased simulator fidelity, and consequently a better training environment for pilots. For flight control systems this results in a more accurate reference signal tracking performance, and an increased tolerance to damage events. Ultimately, high quality aerodynamic models have an important societal relevance by benefiting flight safety. The societal relevance of accurate aerodynamic models, together with the technical challenge of their identification from flight data, presents the ideal case for demonstrating the utility of the new methodology proposed in this thesis.

Two identification experiments in the field of aerodynamic mode identification were conducted with the new methodology. The first experiment was the identification of an aerodynamic model for the F-16 fighter aircraft using a NASA wind tunnel dataset. The internal structure of this wind tunnel model was known, and as such it provided a controlled environment for testing and validating the new methodology.

In the second identification experiment a complete set of aerodynamic models for the Cessna Citation II laboratory aircraft were identified using flight data obtained during seven test flights conducted between 2006 and 2010. In total, 247 flight test maneuvers were flown which together provided a significant coverage of the flight envelope of the Citation II. The complete identification dataset consisted of millions of measurements on more than sixty flight parameters. For this real-life experiment it was necessary to consider the aspects of model structure selection, parameter estimation, and model validation. The geometric model structure selection was performed using a novel approach based on the occurrence of hysteresis in the time trace of the aerodynamic force and moment coefficients. Using the hysteresis analysis method, a number of candidate dimension sets was defined. For each candidate dimension set, a triangulation of the hypercube was created that minimally envelopes the flight test data. The polynomial model structure was selected by validating the performance of a number of prototype simplex spline functions of different degree and continuity order on the hypercube triangulation. More than 2000 prototype spline models were identified using a newly developed, highly optimized software implementation of the simplex spline identification algorithm. The final geometric and polynomial model structures were selected based on the further optimization of the best performing prototype model.

The identified simplex spline based aerodynamic models are phenomenological models, that is, models that are based directly on observational data. Using the developed methods for simplex spline model validation it is proved that the models are both accurate and of guaranteed numerical stability inside the spline domain. The identification and validation results of the simplex spline models were compared with those of ordinary polynomial models identified using standard identification methods. These results showed that the multivariate simplex spline based aerodynamic models were of significantly higher quality than the aerodynamic models based on ordinary polynomials.

The research performed in the framework of this thesis leads to three principal recommendations. First, it was found that the greatest practical limit in the application of multivariate simplex splines in real life data approximation is per-simplex data coverage. To alleviate this problem, and further improve the practical utility of the simplex splines, a software tool should be developed for checking, in real time, the coverage of the system operating domain with measurements. In the case of aerodynamic model identification, such a software tool would provide cues to the pilots for executing specific maneuvers. The second recommendation is that a general triangulation optimization method should be developed that is specifically suited for system identification with simplex splines. Such a method could be based on the global formulation of B-form polynomials provided in this thesis, and would close an important gap in current simplex spline theory. A final recommendation is a real-life implementation of an adaptive model based control system which employs the recursive B-coefficient estimator introduced in this thesis. For aerospace applications, this would result in a fault tolerant flight control system with a built in flight envelope prediction functionality. Installed in future aircraft, simplex spline adaptive model-based flight controllers could increase flight safety by turning catastrophic events into survivable incidents thereby saving human lives.

Contents

Appendix A

Reference Frame Definitions

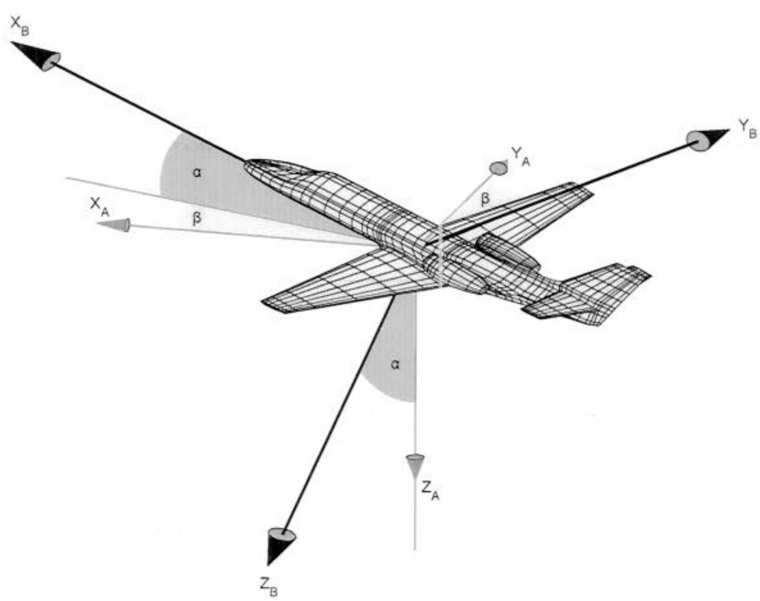

Figure A.1: The body fixed reference frame F_B and the aerodynamic reference frame F_A.

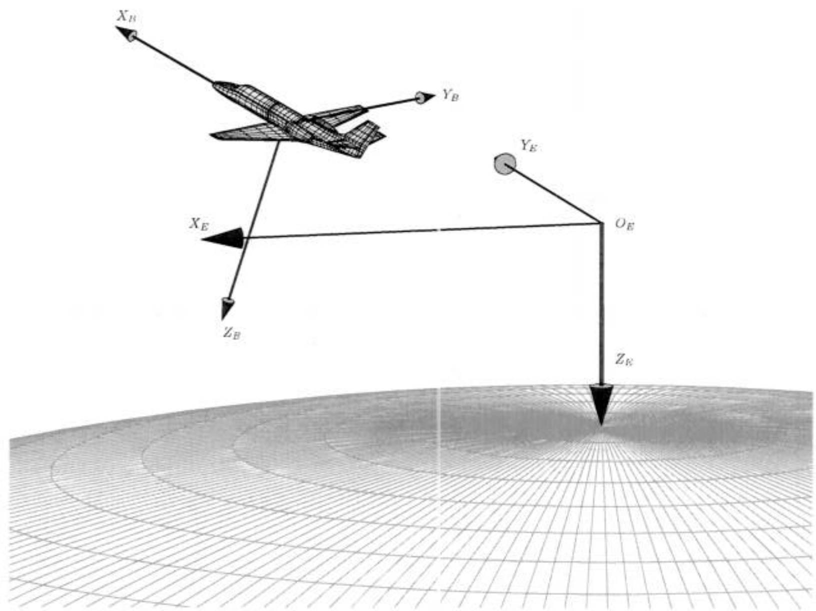

Figure A.2: The Earth fixed reference frame F_E and the body fixed reference frame F_B.

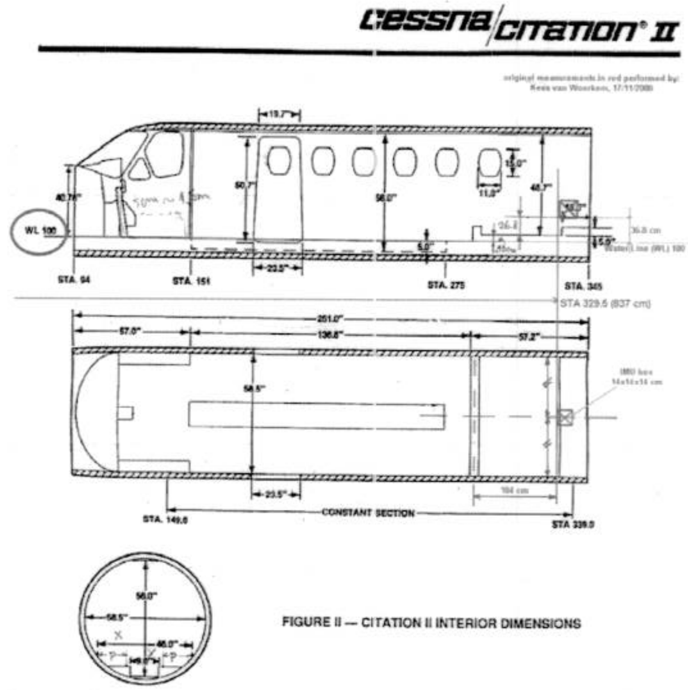

Figure A.3: Location of the IMU box in terms of the aircraft reference frame.

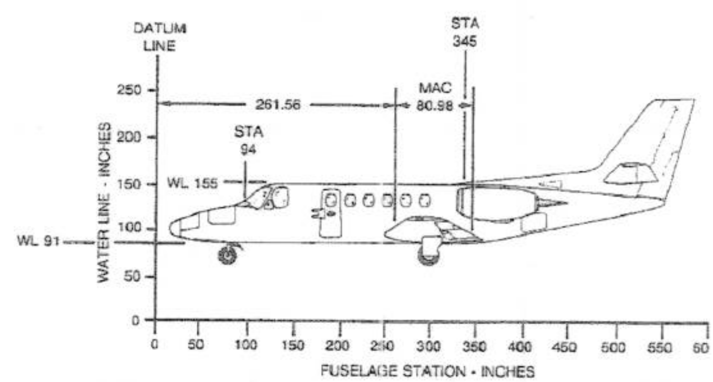

Figure A.4: Definition of the aircraft reference frame of the Cessna Citation II.

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