Oxygen Anomalies in the Aftermath of the BP Oil Spill

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ABSTRACT

The Macondo oil spill in 2010 injected an unprecedented amount of hydrocarbons into Gulf of Mexico deep waters. A poorly constrained fraction of this reduced carbon input dispersed in the water column. As the biological breakdown of reduced hydrocarbons consumes O_2, depletion of this electron acceptor may indicate the past presence of hydrocarbons. Here, we present preliminary results to quantify O_2 anomalies from a sparse data set using bivariate splines.

INTRODUCTION

The BP oil in the Gulf of Mexico lasted 87 days and has caused millions of barrels of oil and natural gas to gush into the ocean at a water depth of approximately 1500 m. For a number of societal and environmental reasons it is important to have an accurate estimate of the amount of oil leaked into the Gulf of Mexico. Governmental labs and private researchers have generated many estimates based on the size and thickness of the oil slicks floating on the surface of the ocean, the flow rate at the riser, laboratory simulation based on the video images of the leaking, and etc. (cf. [9]). According to [8], based on pressure measurements recorded as the valve was closed, Department of Energy labs estimated the flow from the Macondo well to be 53,000 barrel per day (BPD) and a total release of 4.9 million barrels of oil from the BP oil well during the disaster.

The fraction of oil remaining in the water in particular is a critical component because of its potential long-term effect and impact on the Gulf of Mexico ecosystem. The breakdown of reduced compounds, including hydrocarbons, leads to a consumption of O_2. Thus, O_2 levels measured in the water column are indicative of such decomposition processes. In the aftermath of the oil spill, several cruises recorded O_2 concentration profiles in the Macondo area. We present here our preliminary analysis of the data collected on NOAA ship Pisces Cruise IV between August 19 and September 2, 2010, predominantly southwest of the Deepwater Horizon drill site ([7]). From these observational data, O_2 anomalies were quantified by manually curating measured profiles and identifying O_2 depletion against background concentrations in approximately 1 m depth intervals between 700 and 1300 m water depth. To fill in gaps in this large yet sparse data set, we use bivariate splines to constrain the total amount of the oxygen depletion in late August in the area covered on this cruise. The locations where water column profiles were recorded are shown in Fig. 1.

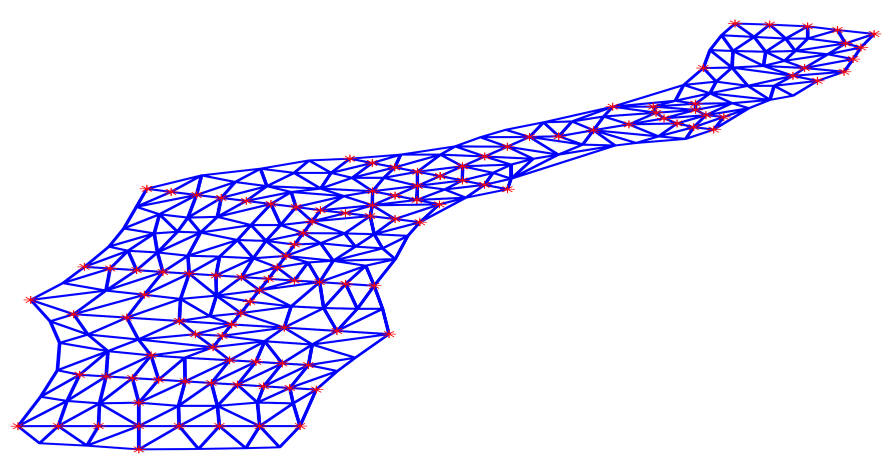

Fig. 1. Locations (in red crosses) where Oxygen Concentrations Were Measured and a Triangulation with Added Locations with No Measured Values

APPROACH

Our computation of the total amount of O_2 depletion from the data collected over the two-week period of Pisces cruise IV entails

1. Computation of a bivariate spline fitting function over the given data values for each of 607 depth layers.
2. Integration of each fitting function across the domain and then summation over the contribution from each layer.

First, by manually adding more locations (although ones where no water column profiles were measured), we create the triangulation shown in Fig. 1 which is a well-conditioned triangulation covered the locations where data values were measured.

Let $S_d^r(\triangle)$ be the spline space of degree d, smoothness $r \geq 1$ with $d > r$ over triangulation $\triangle$. For example, $d = 5$ and $r = 1$. We shall compute $S_j \in S_d^r(\triangle)$ such that

$$S_j = \arg \min_{s \in S_d^r(\triangle)} \begin{cases} E_2(s), & s(x_i, y_i) = o_{i,j}, i = 1, \cdots, 131 \\ & s(x,y) \geq 0, (x,y) \in \Omega, \end{cases} \tag{1}$$

where $\Omega = \cup_{T \in \triangle} T$ and 131 is the number of stations of the research vessel. In terms of B-coefficients, we can write

$$s(x,y) = \sum_{i+j+k=d} c_{ij}^T B_{ijk}, \quad \text{if } (x,y) \in T \in \triangle \tag{2}$$

whose coefficient vector $\mathbf{c} = (c_{ijk}^T, i+j+k = d, T \in \triangle)$ of size $DN \times 1$ satisfies smoothness conditions $H\mathbf{c} = 0$(cf. [5]). Letting $I\mathbf{c} = \mathbf{o}_j$ be the linear equations for the interpolation conditions in (1) with $\mathbf{o}_j = (o_{1,j}, \cdots, o_{131,j})$, we can rewrite the constrained interpolation problem (1) as

$$\min\{\mathbf{c}^\top E\mathbf{c}, \quad H\mathbf{c} = 0, I\mathbf{c} = \mathbf{o}_j, \mathbf{c} \geq 0\}. \tag{3}$$

where E is the symmetric and nonnegative definite matrix associated with the energy functional, i.e., $\mathbf{c}^\top E\mathbf{c} = E_2(s)$. To solve (3), we shall use the Uzawa algorithm (cf.[2]).

Algorithm 1.1 *Start with an initial guess S^0 and parameter vector $\lambda^{(0)} = 0$. For $k \geq 1$, we minimize the quadratic function*

$$\min_{v \in \mathbb{R}^n} \mathbf{c}^\top E\mathbf{c} + \alpha\|H\mathbf{c}\|_2^2 + \beta\|I\mathbf{c} - \mathbf{o}_j\|_2^2 + (\lambda^{(k)})^\top \mathbf{c} \tag{4}$$

to find $\mathbf{c}^{(k)}$ and update

$$\lambda^{(k+1)} = \max\{\lambda^{(k)} + \rho(\mathbf{c}^{(k)}), 0\}, \tag{5}$$

for $k = 0, 1, 2, \cdots$, where $\rho > 0$ is a step size.

Based on the theory in [2], we know this iterative algorithm will converge.

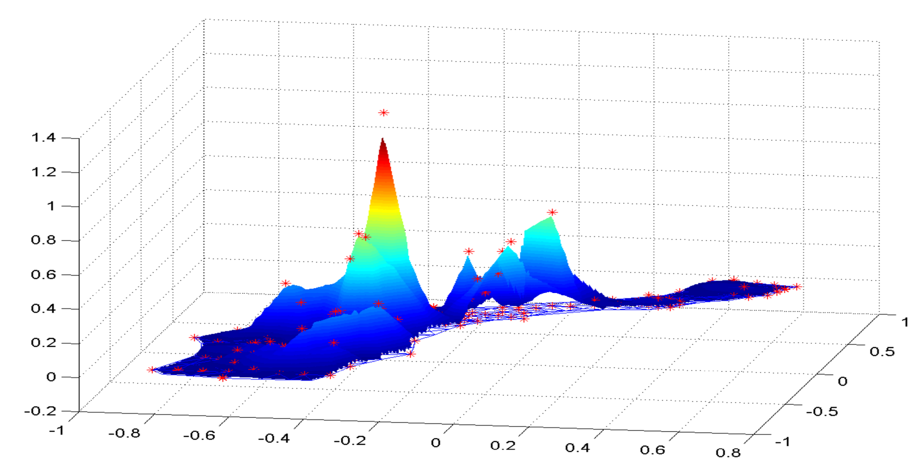

Fig. 2. A Spline Distribution of Oxygen Depletion at a Water Depth of 1114m. Red Crosses are Observed Data Values.

NUMERICAL RESULTS

We implemented this algorithm in MATLAB based on [1] and are able to find a spline surface fitting to the given data values at each ocean depth level. See an example in Fig. 2.

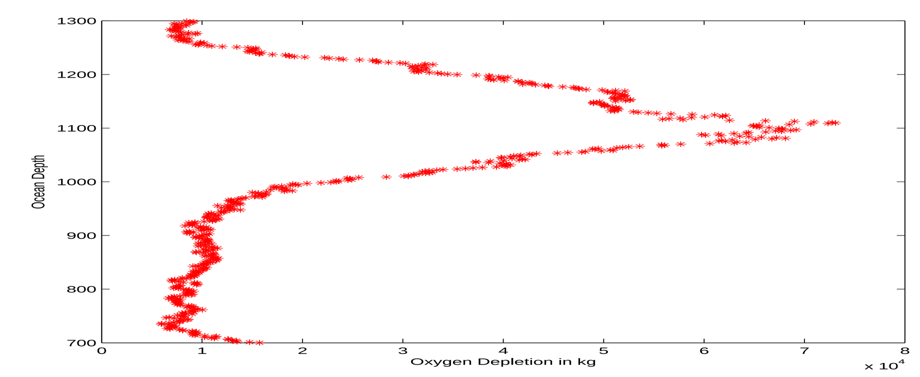

Fig. 3. Oxygen Depletion (in kg per 1m-layer) over 700 m–1300 m below the Ocean Surface.

CONCLUSION AND REMARKS

Conclusion: We present an attempt to compute O_2 depletion in the aftermath of the BP oil spill. Interpolation functions accurately reproduce the observations, and are chosen such that no significant artificial positive anomalies (O_2 concentrations greater than background) are produced. Preliminary estimates suggest an O_2 deficit in the domain covered by Pisces IV of 4.53×10^9 kg, compared to $0.96 - 1.25 \times 10^9$ kg reported in [4]. This difference highlights the significant uncertainty associated with the interpretation of a large but sparse data set and emphasizes the need for a more comprehensive assessment that integrates an analysis of physical forces with state-of-the-art interpolation approaches.

Remark 1.1 *The current spline interpolation balances smoothing with fit to the data. However, smoothing physically represents the effect of mixing processes, and no attempt to establish an explicit link between mixing intensity or the statistics of observed anomalies and the extent of smoothing has yet been made.*

Remark 1.2 *To obtain more accurate estimate, we need to use trivariate splines ([1] and [5]). Due to the dataset size, we will use the approach in [6] to carry out the study.*

References

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